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TLRI Mathematics Enhancement Project: Professional Development Research

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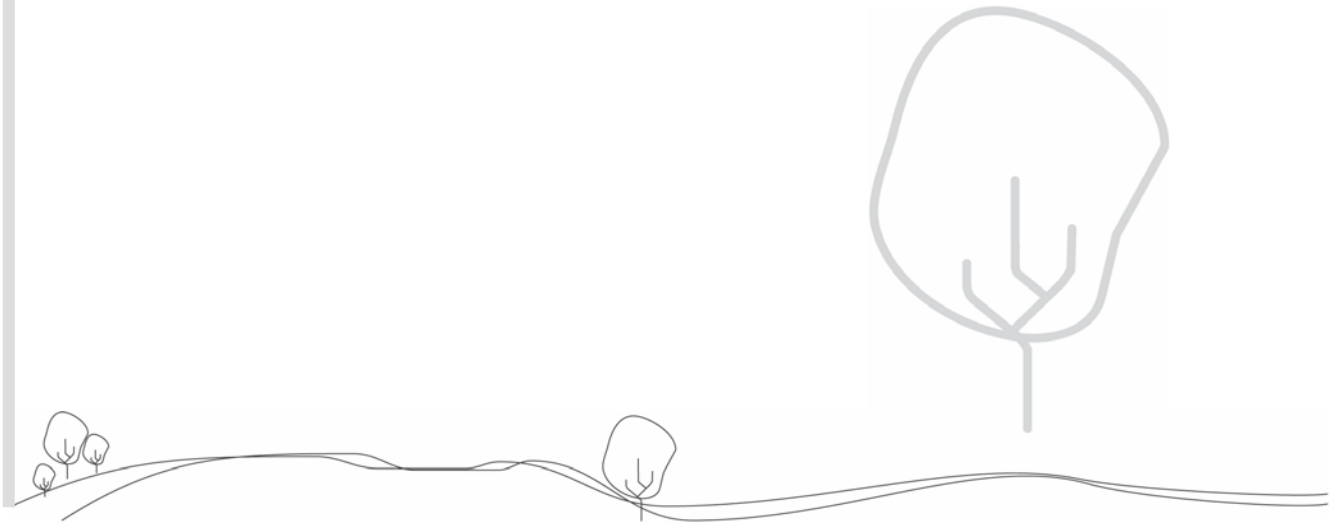

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Mathematics Enhancement Project: Professional Development Research

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Table of Contents

Figures

Appendices

1. Introduction

The research project on professional development reported here was conducted within the context of the Mathematics Enhancement Project. The Mathematics Enhancement Project involves the Mathematics Education Unit of the Department of Mathematics, University of Auckland, working with senior mathematics teachers in low decile secondary schools in the Manukau region. It was the result of a needs analysis conducted in 1999–2000 and is an ongoing development and research endeavour that aims to enhance mathematics achievement at a time when students are making the transition from secondary school to tertiary education. The project works at the student level, at the teacher level, and at the school and parent community level. The intention is to create a mode of professional development for these schools that can be used elsewhere in New Zealand, is realistic in terms of cost and resource input, and will increase the participation of students in tertiary education courses with mathematical requirements. The project currently involves eight Manukau schools. All these schools have high proportions of Māori, Pasifika, and immigrant or refugee students.

The professional development research project discussed in this report was funded by the Teaching and Learning Research Initiative (TLRI), and was situated within the teacher development component of the Mathematics Enhancement Project during the years 2004–2005. Our preliminary research had confirmed other studies that found professional development of teachers requires their active participation in investigating aspects of their practice in ways that take account of the systemic problems of their particular environment (in this case, low-decile schools). The project, therefore, set out to establish whether and how teacher research could form part of effective professional development.

The overall plan was to involve 16 teachers in an ongoing research community that produced quality research on the mathematics learning of their classes. This was to be done by creating four research teams, each led by a university researcher. De facto, the research group meetings were to be professional development sites where best mathematics teaching practice was discussed and support given for classroom changes. The teachers were to be inducted into critical research processes and thus gain insights into their practice as part of a professional community that included research as part of professional practice. The whole process was to be researched as effective professional development.

In the initial proposal the four studies were to be: (university leader; international adviser):

- professional development through further mathematics learning (Judy Paterson; Professor Deborah Ball, University of Michigan);

- how to mentor mathematics teachers in their classrooms (Barbara Kensington-Miller; Professor Anne Watson, Oxford University);
- the development of students' identities as mathematical learners (Dr Hannah Bartholomew; Professor Stephen Lerman, South Bank University);
- using the concept of the didactic contract to investigate and develop best practice (Associate Professor Bill Barton; Professor Colette Laborde, Grenoble University).

In the event, there were seven studies, and a total of 27 teachers involved directly over the two-year period, with other members of the mathematics department of each school occasionally attending the meetings.

Before the project started the initial four studies were supplemented by two further studies as a result of university researchers, Garry Nathan and Viliami Latu, joining the team. These two studies were:

- the development of conceptions of calculus (Garry Nathan);
- Pasifika languages and mathematics (Viliami Latu).

This last study began as part of a separate TLRI project and was reported on at the end of 2004. It continued into 2005 as part of this project.

In the first year, the participating teachers were grouped into these six pre-determined research studies as research partners.

In the second year, an alternative approach was used where each teacher was assisted to develop classroom-based studies with their own question about their own practice. The didactic contract and conceptions of calculus studies finished at the end of 2004, the others continuing through the involvement of the university researchers and a few of the teacher–researchers. The seventh study, on the nature of whole-class discussion, started, led by Suzanne Kerr, one of the teachers now on a full-time Ministry of Education Study Award in the Mathematics Education Unit. One school withdrew from the project as it was overburdened with other research and development projects that also involved its mathematics teachers.

The project was primarily undertaken during a series of teacher meetings. There were 14 of these in total over the two years. In addition the university researchers made several visits to the classrooms of some of those teachers who were part of their team. The university researchers also met regularly (fortnightly in the first year, less regularly in the second) to discuss the progress of the project.

We were fortunate during the course of this research to have visits to the Mathematics Education Unit by four overseas researchers who took an interest in the project. Professor Colette Laborde visited early in the project and was present for the launch, as was Dr Judy Mousley. Professor Laborde was also here for a week at the end of the project. Professor Stephen Lerman visited in early 2005, and Dr Hannah Bartholomew consulted with him when she was on a trip to United Kingdom. Professor Deborah Ball's visit was delayed until 2006 and so was not part of this

report. Professor Marcelo Borba from Brazil became involved during a visit in July, 2005, and Dr Anne Watson visited in the latter stages of the project. In addition, Dr Andy Begg had an honorary position in the Mathematics Education Unit in 2005 and joined the university team as an adviser. Each adviser took time to visit schools or attend teacher meetings. They fed back to us their reflections and ideas at the university researcher meetings.

The research project is reported on three levels: first on the use of research as a professional development strategy; second on the seven organised research studies; and third on the mathematics classroom studies investigated by individual teachers.

2. Project Aims and Objectives

Relationship to TLRI Principles

The professional development research project relates directly to the TLRI Principles. Principle 1 concerns strategic value to education in New Zealand. Our research responds to equity in the New Zealand education system, by specifically addressing the situation for decile 1 and 2 schools in a predominantly Māori, Pasifika, and immigrant region. It directly investigates the processes of teaching and teacher development.

Principles 2, 3 and 4 relate to the quality of the research. By building on work of experienced researchers in a team working in a long-term project, our project was responding to gaps identified in a needs analysis that included a review of previous research. Research processes were already in place. The theoretical frameworks used are internationally proven and the project work has been reported in peer-reviewed international forums. The potential to build capacity amongst the teachers involved was very high. The project was deliberately designed to draw existing teachers into a process of ongoing development through university study to upgrade their qualifications and theoretical and research experience.

Principles 5 and 6 relate to the role of teachers as researchers. This was the explicit topic of the project, and much of the work transferred directly to the learning environment. The overall impact was to create a partnership in which teachers were encouraged to develop and extend their practice

Specific Aims and Objectives

The aim of the overall Mathematics Enhancement Project is to enhance the participation and achievement of senior secondary students in low-decile schools as they make the transition to tertiary education. The intention is to establish a model of development for similar schools in New Zealand.

Principal Aim

The principal aim of our research project was to investigate the effectiveness of research activity as a professional development strategy for senior mathematics teachers in low-decile schools. The objectives were to:

- involve all project teachers in secondary research studies as active participants;
- research the barriers and effective motivators for these teachers' involvement;
- research the effect of involvement on the teaching practices of each teacher, and on the community of practice of the project teachers as a whole.

Secondary Aims

The research incorporated seven specific studies (the secondary research studies), each with its own aims and objectives. These aims and objectives of these secondary research studies were as follows.

Study 1: to research further mathematics learning as a professional development activity. The objectives were to:

- establish a programme of teacher meetings based on mathematical learning;
- research teacher discussion of mathematics learning based on the above meetings;
- evaluate the programme through teacher feedback.

Study 2: to develop and research effective mentoring of senior secondary mathematics teachers. The objectives were to:

- develop a mentoring programme with four teachers in project schools;
- monitor the mentoring programme by teacher feedback and mentor records;
- develop and implement a classroom observation schedule.

Study 3: to research the way that students come to see themselves (and act effectively) as mathematical learners. The objectives were to:

- create case-studies of students' development of mathematics learners;
- undertake comprehensive surveys of students in project schools;
- design a programme for creating a "mathematics learner" environment.

Study 4: to research the nature and formation of implicit and explicit agreements about mathematics learning that exist within senior secondary mathematics classes in low socioeconomic schools. The objectives were to:

- undertake discussions in the classroom to outline the didactic contract;
- make classroom observations of the didactic contract and breaks in its operation;
- research the use of this concept as a basis for development of teachers.

Study 5: to investigate the way that students' conceptions of calculus change as they move from secondary to tertiary mathematics education. The objectives were to:

- undertake classroom observations to determine student conceptions of calculus in secondary and beginning tertiary situations;
- interview students to explore further the nature and basis of change for these conceptions.

Study 6: to investigate the use, understanding, and learning potential of Samoan and Tongan mathematical discourse in the classroom. The objectives were to:

- develop and administer a questionnaire that would explore students' understanding of mathematical discourse in English and their own Pasifika language;
- investigate the potential of Samoan and Tongan languages for new mathematical vocabulary development.

Study 7: to investigate the features of whole-class discussion in mathematics, particularly those aspects over which the teacher has control. The objectives were to:

- observe whole-class discussion in the classroom to describe its features;
- interview teachers about their whole-class discussions and their use of them for teaching mathematics.

Tertiary Aim

The research further involved a number of teachers developing their own classroom-based studies. The overall aim was to establish classroom studies that were practicable for participating teachers and would address a feature of their own classroom practice. Specifically, the objectives were to:

- identify an aspect of practice directly related to mathematics learning that was causing concern;
- establish an observation, and possibly an intervention, that would lead to the teacher having a better understanding of that practice.

3. Research Questions

We know that professional development needs the active involvement of teachers (Britt et al., 1993; Begg, 1994), and there is much literature about teachers as researchers (e.g. Zeicher, 2003). Our project, however, is situated in a particular group of low-decile schools which have unique demands upon teachers who already have unique needs (Kensington-Miller, 2002). We are concerned, therefore, with the question of the efficacy and appropriateness of research as a means to professional development in such situations. Therefore the first research question is as follows.

1. Does involvement in classroom research lead to positive changes in teacher behaviour and classroom practice for senior mathematics teachers in low-decile schools?

The project was conceived as consisting of four (subsequently seven) pre-determined studies which teachers would join as active participants. The research questions for these seven studies were as follows (all questions include implicitly the low-decile situation).

2a. Will teachers' participation in new mathematics learning cause them to reflect upon their own classroom practice and lead to changed teaching behaviour?

2b. How can peer mentoring be established between teachers in participating schools, and how will any mentoring established affect their practice?

2c. How do students come to see themselves as effective mathematics learners, and how do teachers come to see themselves as effective mathematics teachers?

2d. What agreements are formed between teachers and students regarding mathematics learning in the classroom, and how are these established?

2e. What are students' conceptions of calculus at senior school level, and how do these compare with their conceptions at university?

2f. What use is made of Tongan and Samoan language in the classroom amongst speakers of those languages, and can an understandable mathematical discourse be established?

2g. What characterises whole-class discussion in senior mathematics classes, and what are its effects on student learning?

In addition to these explicit questions, some teachers developed their own smaller studies. In general these were not formalised, nor did they get to the stage of formal reporting beyond verbal accounts of their investigations to the teacher meetings. The questions investigated all fall under the same umbrella question.

3. What aspect of my classroom mathematical practice might I change in order to improve student learning, and how could I achieve this change?

4. Research Design

The Research Team

The project was developed during 2003 with a small group of university researchers and teachers within the Mathematics Enhancement Project. Responsibility for the design was taken by Dr Hannah Bartholomew and Associate Professor Bill Barton, with assistance from Barbara Kensington-Miller and Judy Paterson.

The study involved a research team of six university researchers of varying experience, from an associate professor to masters-level researchers. All bar one had had secondary mathematics teaching experience; the other had primary experience. This was supplemented by the 27 teacher–researchers, of whom eight had some experience at masters-level research studies either in New Zealand or overseas.

The university and teacher groups were developed from those who had already been involved in the Mathematics Enhancement Project. However, for the project reported here, four new schools (about 12 teachers) and two university researchers joined the group for the first time. The list at the beginning of this report shows all those who were involved at some stage of the research. In the first year (2004), Associate Professor Bill Barton directed the project, but in the second year he took a more minor role owing to other duties. Dr Hannah Bartholomew took over direction, and Suzanne Kerr from Tamaki College became a university researcher as she had a Ministry of Education study award for that year.

Ethics

Ethics approval was granted through the University of Auckland Human Participants Ethics Committee (Reference 2003/391). This involved permission by the principal of each participating school, and each of the teachers involved.

Research Participants

The participants in this study were the teacher–researchers themselves. All were teachers of Year 12 or Year 13 mathematics with statistics or mathematics with calculus classes in decile 1 or 2 schools in the Manukau region. Of the 27 teachers who became involved at one stage or another,

20 were educated, trained, or had taught outside New Zealand (10 were recent immigrants); 12 had a language other than English as their first language; 12 had mathematics majors in their degree; 4 had no university mathematics background; and 14 moved to the project school or away from it in the two-year period of the study.

In summary, the group was varied in cultural background, language background, and mathematical training. It was transient, and represented the full age-range from first-year teachers to those with 30-plus years' experience, with two teachers having remained in the same position for more than 10 years.

Methodological Approaches

The overall task was to involve teachers in research as active members of a professional team under university leadership. The overarching methodology was thus naturalistic, with the research team as a whole also being the participants. Hence participant observation was the dominant mode.

However, it needs to be clear that large parts of the research, including the overall research question, was primarily driven through the university researchers. While they had a perspective that could be distinguished from that of the teacher–researchers, they were also participants in the research effort. The participant observation was, therefore, biased in its view of the project. The effect was ameliorated, first, by the involvement of teachers in the research as commentators and contributors to the design and, second, by the university researchers being joined in the second year by one of the teachers on study leave.

The primary research project and the seven secondary research studies used mainly qualitative methods, predominantly classroom observations, interviews, and written feedback from the teachers and research leaders. A second emphasis running through all studies was that of triangulation. Classrooms and teacher development are notoriously complex environments to research, and thus confirmation of research results is vital. This was achieved by the sharing of results from different parts of the studies. At the university meetings, we discussed results that were relevant to each others' studies, and shared hard data if appropriate.

Each of the studies operated separately under its own methodology, as outlined in the individual study reports in the appendices.

The classroom studies undertaken by the teachers themselves can be characterised as action research-like methodology, but they did not reach any formal stage.

Research Process

Schools were invited to participate in the Mathematics Enhancement Project (and thus, for 2004–05, in this project) through meetings with the principal and the heads of the departments for mathematics (HoDs mathematics), held in August and September 2003. A meeting of all the teachers participating at that stage was held in November, 2003 at which the six initial studies were outlined, and teachers invited to join one of the teams. In December, the project was launched at a ceremony and one-day seminar at the University of Auckland, involving the Mayor of Manukau City, Sir Barry Curtis; the Vice-Chancellor, Dr John Hood; and two of our overseas advisers, Professor Colette Laborde and Dr Judy Mousley. Presentations were given by Professor Graham Smith (University of Auckland), Associate Professor Megan Clark (Victoria University), Mr Robin Staples (Principal, Hillary Collegiate), Suzanne Kerr (Tamaki College), Vesi Talamaivao (Western Springs College) and the two overseas visitors.

The research team (university researchers and teachers) met regularly over the two-year period. Most meetings in the first year included a general discussion period, a visiting speaker mathematics presentation and discussion, and group meetings of the research teams. Most meetings were three-hour afternoon meetings on school days at Manukau Institute of Technology.

In the second year, the meetings changed to weekend and holiday times at the teachers' request, prompted by the difficulty of finding adequate replacement teachers when meetings happened in school time. The style of the meetings had also changed, with more full-group discussion of teachers' own studies of their practice, and some workshop-type activities with a curriculum focus (again at the teachers' request).

The series of meetings is listed below.

Meeting (i)	16 September 2003. Anticipatory and planning meeting with HoDs mathematics of the participating schools.
Meeting (ii)	20 November 2003. Anticipatory of the contract. Information meeting for teachers to describe project and answer questions.
Meeting (iii)	10 December 2003. Project launch.
Meeting 1	28 January 2004. Initial meeting of university researchers and teachers.
Meeting 2	16 March 2004. Dr Paul Hafner discussed the degree-diameter problem from network theory.
Meeting 3	4 May 2004. Dr David McIntyre analysed the mathematics of yacht racing.
Meeting 4	16 June 2004. Dr Brian McArdle presented findings from a study to model the distribution of clams in an estuary.

Meeting 5 — 6 August 2004. Jerry Lane, a teacher from Mangere College who had been on a RSNZ fellowship in 2003, gave a talk on the mathematics of traffic flow and standing waves in particular.

Meeting 6 — 9 September 2004. Dr Sina Greenwood gave a presentation on topology and Brunnian Rings. A Curriculum workshop was held including a presentation by Barbara Kensington-Miller on work she had done at a pedagogical content seminar with Anne Watson (Oxford) and John Mason (Open University) in United Kingdom in July, and a discussion led by Judy Paterson on the power of considering doing and undoing in mathematics.

Meeting 7 — 30 September 2004. Full-day meeting. Dr Rachel Fewster discussed the problem of calculating the probability, based on Bayesian theory, of islands becoming re-infested with rats after they have been cleared. A curriculum workshop on simulations was held and teachers discussed their learning over the period of the project.

Meeting 8 — 13 November 2004. Held at the University of Auckland. Vesi Talamaivao, a teacher from Western Springs College who had been on a RSNZ fellowship in 2004, presented his work on the mathematics of sport.

Meeting 9 — 24 February 2005. First meeting for 2005. Organisational.

Meeting 10 — 28 April 2005. Full-day meeting. Dr Jamie Sneddon presented the first of two talks on using matrix multiplication to analyse tournaments.

Meeting 11 — 8 June 2005. The second of Dr Jamie Sneddon's two talks.

Meeting 12 — 21 July 2005. Full day meeting. Alenka Klokocovnik, a high school teacher from Romania, spoke about using paper folding to create geometrical constructions, and Jonathan Baxter, an artist who has been working with origami for 20 years, spoke about using paper to create two- and three-dimensional shapes.

Meeting 13 — 30 August 2005. Project feedback session.

Meeting 14 — 6 October 2005. Final full-day meeting. Anna Dumnov, a teacher from Senior College of New Zealand on a Royal Society of New Zealand Fellowship, presented findings on the history and uses of complex numbers. Three teachers presented the findings of their individual studies.

A feature of the project was the turnover of teachers—a situation that was exacerbated by a turnover of principals and HoDs mathematics. Three schools had new principals in the first year of the project, and four of the schools had new HoDs mathematics. Every school had at least one teacher leave or a new one enter the project, and some had two. These changes were disruptive to an ongoing teacher development programme, and also placed considerable stress on participants.

Replacement staff were difficult to obtain, and all other teachers were involved in filling temporary gaps in teaching staff.

A more positive development was the enrolment of six teachers in a masters programme in mathematics education as a result of their involvement in the project. Teachers were also supported through the project to apply for Ministry of Education Study Awards and Royal Society of New Zealand Teacher Fellowships. Two study awards were obtained for 2005 and one fellowship for 2006.

One school withdrew from the project in the second year. Its HoD mathematics felt that its involvement in two other major projects meant that an unfair burden would be placed on their teachers. These other projects were cross-curricular and therefore took precedence. In response, we increased the number of teachers from the other seven schools rather than initiate another school in the middle of the project.

The design of the teacher research programme changed in 2005. The project was initiated with six research components, with teachers encouraged to join one of these groups. However two of these studies (those of Bill Barton and Garry Nathan) finished, and the studies led by Barbara Kensington-Miller and Judy Paterson had entered a phase where teacher involvement as researchers was not so appropriate.

More critically for the project, the teachers now had some ideas about what classroom research entailed, and had developed their own interests. Therefore it was decided to give them more freedom to design their own classroom-based study, and to cluster them in various ways for research guidance and reporting. It was expected that these studies would be learning experiences for individual teachers and not of general applicability.

This new model for the teacher research component of the project included time for teachers to report and discuss their research activities in small groups at the teacher meetings. Each group was facilitated by a university researcher whose role was to take notes, and to move the research forward by raising questions and offering advice on translating research ideas into manageable tasks.

Three of the classroom studies progressed to a reporting stage. They were concerned: the impact of homework on attainment; the factors that cause good students to abandon their studies; and the effect of teacher pausing in classroom discussion.

Towards the end of the second year eleven teachers were interviewed on top of the reporting undertaken and recorded during the teacher meetings.

Contribution of Advisers

Associate Professor Andy Begg, a New Zealand mathematics educator who had just returned from two years at the Open University in the United Kingdom, held an honorary position in the Mathematics Education Unit in 2005. He attended all the university team meetings in that time, acting as a critic of what we were doing and offering suggestions for improvements. Dr Begg is known for his advocacy of the independence and competence of teachers, and he challenged us several times on the differing roles of the university and teacher researchers. His input was at a day-to-day level and was much appreciated. He also acted as an extra mentor for the four researchers undertaking these studies as part of their doctorates or masters degrees.

Professor Marcelo Borba, State University of Sao Paulo, Rio Claro, Brazil, visited the Mathematics Education Unit in July and August 2005. As part of his visit, he reflected on the project. His input was especially valuable in his consideration of language and cultural issues.

Professor Colette Laborde, Grenoble University, France, was in Auckland both at the beginning and at the end of the project. Her presence at the opening ceremony and presentation to us on didactic contracts was especially useful, as was her perspective on the project from a French point of view, where mathematical formalism and mathematical knowledge of teachers is much more taken for granted.

Professor Steven Lerman, South Bank University, England, was visited during 2004 by Dr Hannah Bartholomew, and then he visited Auckland in the early part of 2005. His main contribution was to advise Dr Bartholomew on issues concerned with identity and communities of practice.

Dr Judy Mousley, Deakin University, Australia, was only present in the lead-up to the project and for the opening. However her comments in those formative stages were useful as she has considerable experience in teacher development and relationships between university researchers and teachers as researchers.

Dr Anne Watson, Oxford University, England, was visited by Barbara Kensington-Miller in May and June 2005 and then came to Auckland in September that year. Her main contribution was as an adviser to Barbara Kensington-Miller on issues of mentoring and teacher development. However she also had pertinent things to say about mathematical tasks for teachers, and Barbara Kensington-Miller used many ideas from Dr Watson's workshops in England in the teacher meetings.

Professor Deborah Ball did not visit New Zealand until 2006. Thus her contribution has been most useful for the next stages of the Mathematics Enhancement Project not covered by this report.

5. Project Findings

We report first on the substantive research question and then on the findings of the individual studies.

Professional Development Through Research

The principal aim for this project was to investigate the effectiveness of research activity as a professional development strategy for senior mathematics teachers in low-decile schools.

The objectives were to:

- involve all project teachers in secondary research projects as active participants;
- research the barriers and effective motivators for these teachers' involvement;
- research the effect of involvement on the teaching practices of each teacher, and on the community of practice of the project teachers as a whole.

The project was designed with the intention of a team of university mathematics education researchers leading groups of teacher–researchers working on substantive research topics. Although these studies were more or less successful in themselves (see Findings from individual research studies, p. 15), they were not effective in involving teachers in a substantive way in critical evaluation of mathematics education issues in their classroom. This was largely due to a lack of appreciation of the difference between a researcher's research question and a teacher's research question.

Two of the studies were not suitable for full teacher participation: one (conceptions of calculus) because it was already fully designed, and one (collegial mentoring) because it was too difficult to separate the teachers as subjects and teachers as co-researchers. Another study (professional development through mathematics) allowed some teacher input into the design, but also struggled to separate teachers as subjects and co-researchers. The other three projects were suitable for teacher participation although only one (Pasifika languages and mathematics) fully achieved this.

A further difficulty was that, while researchers understood the need to focus on a small-scale study, many teachers embarked on wide-ranging and unmanageable questions. As researchers, we tried to allow teachers to develop their own questions but also tried to steer them in a direction that related to our original ideas—and managed neither.

Teacher Research in Year 2

In the second year of the project, research was re-conceived from the teachers' point of view Rather than identify topics which teachers could "sign up to", we introduced a model for teacher-led classroom research, and encouraged teachers to define a research topic of their own.

This was more successful, although substantive research was not forthcoming. It is possible that formal research of an action-research style might be forthcoming if more than one year was available, but we feel that this is unlikely. The style of research practicable for these teachers in this context is more like a guided study than formal research. However, the involvement of teachers, and the development of a critical perspective on their practice, did emerge.

The model was a cyclic process, (a modified action-research cycle), whereby teachers were encouraged to try something out to observe its effect, reflect on the implications of the change, and try something else. The focus on an initial action was helpful in generating activity among the teachers, and in keeping projects manageable. Hence even if the overarching topic was a very big question, the first step of the process was to focus in on a concrete change that could be made in teachers' classrooms.

The teachers had a greater sense of ownership of the research than they had in the previous year. Evidence of this was, for example, two of the teachers making extended presentations to the group of their own classroom studies—in the first year, no teacher was in a position to make a presentation on their own. Furthermore, the teachers started bringing written data that they had collected to the meetings—rather than just participating in discussions with their experiences. Furthermore, the research activity was more closely tied to the teachers' own practice. For example, one teacher spoke of the way she changed their verbal input to whole-class discussion by consciously lengthening waiting times, another reported on data collected on the effectiveness of a new homework regime. Yet another teacher, who was investigating students whose achievement faded during the year, moved from being a detached observer of these students to intervening and involving other staff. Thus, though most of the activities in the first year were added on to teachers' existing practice, in the second year they were an integral part of what they were doing in their classroom.

In the first year, the university-based researchers unwittingly made themselves indispensable to the classroom-based research. They undertook to visit teachers in their classrooms, and most of the research activities took place through their lesson observations and subsequent discussions. Time that was allocated for research activities in meetings was often spent on administrative issues such as setting up school visits, and there were severe limits on the amount of time we could give to each teacher.

In the second year, our visits to classrooms were much less frequent. Time was spent in each meeting discussing the research activities that were taking place, and in these the role of the university-based researchers was predominantly to take notes and re-focus discussion.

Participation in a discussion of other people's research was valuable for these teachers. The evidence for this was the change in the second year in the nature of research discussion. It was noticeable even at the beginning of the project that when the teachers met as class-level groups (for example, all Year 13 calculus teachers together), the discussion was collaborative and contributive and aimed at helping everyone improve their teaching. But in research discussions, the teachers stood back and talked about something separated from themselves. In the second year, they participated much more fully. It was notable that some teachers took increasing control over other aspects of the meetings as the second year progressed.

There was a group of teachers who did not significantly engage with the research in either year. This raised questions about accountability in that teachers were being paid to be in the project but were not contributing to it. It also raised the question of the level of the demands that can be made on teachers. These problems were never satisfactorily resolved. Rather than insist that everyone brought something to each meeting, our approach was to treat that which *was* brought as a resource on which we could all reflect, and which other teachers could take and adapt. Perhaps those teachers who did not engage needed longer to develop the confidence to initiate their own research.

Another factor that appears to have had a bearing on the different results between research activity in the two years is the development of the community of practice (see A developing community of practice, p.13). By the second year, the group was much more cohesive. The evidence for this was the quantity and quality of talk in teacher meetings: the proportion of time in which teachers spoke increased, but also the type of talked changed from recounting experiences to discussing how ideas could be adapted to different situations. Other evidence of a forming community of practice was the establishment of inter-school relationships in which contact was made privately outside the project meetings. (These teachers had all met each other prior to the project at NCEA training sessions and the like, but few lasting relationships had formed). In addition, teachers were much more willing to contribute their own ideas; for example, the presentations mentioned above, and an extended contribution on lesson starters. The success of the model adopted in the second year occurred at the same time as the community of practice developed. Where any causal relationship lies cannot be determined, but we believe both that the research activity could not have been successful without the developing community, and that the research activity contributed strongly to this development.

Ownership and Process

One way of interpreting this evidence is that the teachers took ownership of the research activity. The original six projects were pre-determined, and pre-structured. There was always a tension between teachers developing a sense of ownership of the research (both its questions and its conduct), and the university researchers steering the projects to meet external needs (for example, their own masters or doctorate completion). Although the university researchers attempted to involve the teachers as partners (rather than subjects), this only worked in two cases. In both these

cases the teachers became interested in becoming serious researchers outside their normal classroom practice.

A second perspective on the difference between the two approaches is that, in the first year, we began with themes or subjects of attention, and then developed research activity from them. In the second year research processes were of central attention, and teachers' research questions emerged from a combination of subjects of interest and what might be possible in their classroom contexts.

A Developing Community of Practice

One focus for our investigation of research as professional development was the particular features of decile 1 and 2 schools compared with higher decile schools. These features were detailed in Bartholomew, Barton, Kensington-Miller, and Paterson (2005) and can be summarised as:

- poor resourcing, and less well-equipped students;
- higher absentee rates and a lower emphasis on academic subjects;
- more transient, less healthy students with fewer opportunities for homework;
- a higher proportion of students having English as a second language;
- test performance below average, giving teachers little extrinsic affirmation;
- "pyramidal" schools with fewer senior classes, so teachers of senior mathematics classes are isolated and there are fewer role models for junior students;
- few effective relief teachers, particularly in mathematics;
- more students have gaps in their mathematics knowledge;
- a higher proportion of under-qualified mathematics teachers and higher transience of teachers;
- a greater proportion of mathematics teachers were trained in another country, and more of them have English as a second language;
- higher levels of teacher stress, and lower student expectations of achievement and advancement beyond school.

These characteristics make the formation of a community of practice amongst teachers more difficult. When the project started, there was no community of practice despite the fact that four of the schools had been in the Mathematics Enhancement Project for one or more years. For example, many teachers did not know who taught parallel classes in nearby schools and there was no mechanism for updating this information yearly as teaching allocations changed.

However the community developed throughout the project so that, in the final interviews, all teachers spoke positively of the community, even those who had not been regular attendees, including two who had stopped attending altogether. In general this was spoken about more positively than the research activities.

> In meetings we have people coming from all different schools. And we talk about things we do … because otherwise you just feel as though you are a person alone, closed in a room for a long long time, and then you suddenly find there are other people in those types of rooms and you sit down, have a chat. That feeling is really, really good. (MEP teacher)

A measure of the teachers' growing confidence was reflected in their changing approach to research. Although they rejected the pre-organised model of the first year, and turned to individual critical reflections through the second year, at the final meeting they requested that any continued research focus be done in clusters of teachers working together on the same idea. That is, they appeared happy to return to the first-year model, with the difference that they would do the pre-determination. They had the confidence that they could choose suitable topics and conduct research in equal collaboration with university researchers.

Putting the community of practice at the forefront of professional development for these teachers is an acknowledgement of the complexity and variation of the teaching of mathematics in these schools. The systemic issues for these teachers have been highlighted in earlier research associated with the Mathematics Enhancement Project (Kensington-Miller, 2002). Successful professional development must ultimately provide a long-term, sustainable means by which teachers can improve their practice.

The project enabled some theorising of this situation. It was possible to identify two features of teachers' engagement that made reflexive practice[1] more difficult. One feature came to be called "the dragon": this was a fear of being exposed as inadequate as a teacher. The difficulties of these schools made teachers feel close to the edge of failure, and thus any change was a danger. The depth of this fear of being seen as failing was evidenced by an event in late 2006. A teacher who had been part of the project but had spent 2006 in the Mathematics Education Unit as part of the university group nearly did not attend a seminar on the project for fear that she would be exposed as inadequate. In fact, this teacher was one of the most successful teacher–researchers in the group.

The second feature was that of teacher identity. All teachers construct themselves in such a way that they have means of coping when things go wrong, and they are deeply emotionally connected to these constructions. In order to engage in reflexive practice such mechanisms need to be put aside so critical self-examination can take place. Both these features are discussed in depth in Bartholomew et al. (2005) and Barton et al. (2005).

We theorise that herein lies the mechanism of the community of practice. Successful participation in the community is a necessary and sufficient condition for these two features to be put aside.

Finally it is important to note that the development of the community of practice for these teachers is not yet finished. At the end of 2005, responsibility for organising meetings was passed

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[1] Reflexive practice means activities that bend back on to the subject; that is, practice that then affects the practitioner.

to the teachers, and arrangements were made for the first meeting in 2006. The university researchers were to stay involved, but as participants only. In fact no meetings have eventuated although the university team are planning to reinitiate them in Term 3 of 2007.

Conclusions

When this project started, our orientation was that involvement in classroom research of some kind would be the best way for teachers to achieve a critical eye on their practice, and that this should be a major aim of professional development.

The two main findings are that:

- Research can be an effective strategy for professional development, but that the nature of this research will only be effective if it is
  - undertaken from the point of view of the teacher;
  - undertaken at a formality that is commensurate with the experience and intentions of the teacher.
- Involving teachers in any sort of research leading to critical activity is interrelated with the development of a fully functioning community of practice.

Another way of saying this is that the role of research in professional development is that it can enable teachers to be engaged in a different way in the classroom. Formal research is probably only possible if the teacher steps out of that environment (for example, to do further studies). Research has the benefit of empowering teachers to tap into their own interest and enthusiasm.

There is also evidence that the input of external researchers is necessary, but as organisers and research guides, rather than as research leaders.

Furthermore, we now believe that the role of research in professional development is not what we expected. While we still believe that critical classroom activity is a worthwhile objective, we now view the major aim of professional development as the formation of a community of reflexive practice. Indeed, such a community is probably a prerequisite for teachers being critical practitioners and transforming their classroom behaviour. Thus, our finding should be reinterpreted as that research activity is an effective contributor to the development of a community of practice.

It is therefore recommended that professional development initiatives of all kinds be designed to maximise their contribution to a sustained community of reflexive practice; that is, a community that functions on a wider scale than the particular initiative being considered.

Findings from Individual Research Studies

Below are brief summaries of the results of the seven studies conducted as part of the project. Fuller reports of these studies are attached as appendices. The appendices also contain brief accounts of two of the teacher classroom studies.

Professional Development Through Mathematics Learning

The study of professional development through further mathematics learning examined teachers' responses to a model of professional development that was focused on using tertiary-level mathematical input as a stimulus to encourage teachers to reconnect with their mathematical learner selves and in so doing to "re-view" learning. In 2004 and 2005, data were collected from the teachers who attended the workshops at which mathematicians and statisticians presented short talks on topics of their choice. These data were both written and oral. The written data were often in response to prompts designed to encourage the teachers to think about learning (their own and their students') and about teaching (their own and in general), of the mathematical topic being presented in particular. The data from 2004 and early 2005 has been transcribed and subjected to initial analysis.

This is a doctoral study being written up during 2006. Results indicate that the stimulus has been successful in encouraging teachers to identify with the learners in their classrooms and to question aspects of their teaching practice. It has also become clear that many of the teachers have found the mathematical talks and subsequent discussions stimulating and energising. These positive results come from teachers who attended more regularly and who, in general, appear to be more confident, about both their mathematical and linguistic ability in English. Some of the teachers who found the mathematics or the linguistic challenge too great have found reasons not to continue to attend the workshops.

A typology of the teachers responses is being developed in the doctoral study in terms of the connections made between the mathematical stimulus, their own and their students' learning, their feelings about teaching and the action teachers say they intend to take in the classroom. The role played by "teacher energy" appears to be the key in promoting teachers' consideration of change and consequently enhancing the potential effect of the professional development possibilities they are offered.

Collegial Mentoring

In the study of collegial mentoring, three different scenarios of mentoring were explored: teachers within the same school; teachers between schools; and a teacher with a university researcher. The teachers involved were given considerable autonomy and assistance to help establish the mentoring relationships.

For some there were many difficulties in motivating the relationships throughout the school-year period; others met on a regular basis, and some met without any assistance. However, overriding the mentoring was a lack of school support. The findings indicated that for teachers to work together in a one-to-one relationship, they need not only the support from their departments, the administration, and the senior management within their schools, but also from the teaching community to which they belong.

One of the main difficulties was an organisational one with administration in schools. Working with another teacher in the same school and visiting a class was often impossible, as it required relief in the classroom for one partner, which had negative repercussions. Students reacted to a change of teacher, and very often the substitute teacher did not have a background in mathematics. This caused anxiety as precious teaching time was lost. Between schools, timetables varied in structure requiring more time away from classes for a visiting teacher.

There were also hierarchical problems. Power imbalances are unavoidable, but in this study they were reinforced either by the person who was more senior or experienced, or by the less experienced or more junior placing the senior person in an elevated position. Collegial mentoring, which is intended to be equal and complementary, could not happen when the partner with more power critiqued the other, as it created a top-down relationship.

A third area of difficulty was the fear of exposure. The teachers lacked the confidence to trust the other person in their classroom for fear of what they might see or not see, whatever that was. As a result, those of equal status in a mentoring pair found excuses not to meet. Tied up with this were cultural differences in teaching, as the teachers came from a variety of backgrounds, which included Indian, Iraqi, Niuean, Cook Island, English, Welsh, and New Zealand.

As a result of the professional development meetings, a community of practice developed in which teachers began opening up, trust developed, and new dynamics were created between teachers. This resulted in positive spin-offs for the mentoring, as teachers became more confident in working with their colleagues. It was concluded that such communities help hierarchies disappear and allow trust to form, and thus collegial mentoring relationships become easier to establish and develop.

Questions of Identity

The initial aim of this study on the development of students' identities as mathematical learners was not successful for a number of reasons, but the question of identity—in particular teachers' identities—remained a key analytical tool throughout the project.

The topic had the effect of shifting teachers' focus away from their own behaviour and on to that of their students, at times feeding into existing deficit views of students.

However, in the second year of the project some of the teachers who had previously been involved in the identity group continued to develop research questions around this theme, and

clearly found the process personally rewarding. In one case, an interest in the ways in which some students appear to "give up" in Year 13 led to an initiative involving the Year 13 dean and school counsellor in trying to prevent this.

Furthermore, with respect to the teachers themselves, identity proved a useful tool in theorising the range of responses to the project, including some of the resistance we encountered. A key aim of the project was to make a particular kind of professional identity (that of researcher) available for these teachers. The notion of "community of practice" is useful as it makes explicit the identity development that takes place as one becomes a member of the community.

However, a limitation of the model is that it suggests a single route from peripheral to core membership that takes no account of the differences between teachers. Yet we found considerable differences in the ways different teachers engaged with the project, which we believe can only be understood in relation to their identities in a wider sense. By considering the ways in which the biographies of individual teachers intersected with the situations in which they found themselves (in their schools and in project meetings, for example) we were able to identify some common themes. This approach was particularly useful in recognising some of the extreme difficulties that are associated with change and experimentation for teachers in low-decile schools.

Using the Didactic Contract

The intention of this study was to use the concept of didactic contract, as elaborated by the French theorist Brousseau, as a tool for reflection on classroom activities. A didactic contract is an implicit or explicit agreement between teachers and student(s) about an aspect of the mathematical learning process. The study was adopted by a group of three teachers in the project.

The study method was to use the teacher meetings to concentrate on one aspect of the classroom and attempt, in discussion, to frame it using the idea of didactic contract. The teachers' understanding of the didactic contract(s) involved were then tested in the classroom through observations made by the university researcher in the classroom. These observations were then debated for their meaning in terms of the contract in an email dialogue between the university researcher and the individual teacher.

The technique was successful in promoting critical discussion directly in one case, and indirectly for the other two teachers.

In the case that was directly successful, the situation under discussion was whole-class discussion; that is, when the teacher was explaining a mathematical concept to the whole class at once in a way that involved blackboard elaboration, questions, answers, and some student participation at the blackboard. Contracts were identified, particularly surrounding the way questions were to be answered and the ways that students could interact with each other to arrive at answers. Some contracts concerning student-to-student interaction were also noted. This proved to be an excellent basis for subsequent email discussion of the classroom incidents and critical reflection of the teacher's practice.

This case led to this particular teacher following through with whole-class discussion as a research project in the second year under the terms of a Ministry of Education Study Award.

The other two teachers were both concerned about students taking responsibility for learning, particularly with respect to homework. While it did not lead to useful critical discussion in the first year, these two teachers, with their head of department, became a team that took the theme of homework as their self-directed study in the second year. It is difficult to tell whether the framing as a didactic contract helped them to think about the parameters of the second study.

Conceptions of Calculus

This study explored the nature of conceptions of mathematics that students develop as they work with mathematics in increasingly complex ways. Over a year, weekly visits were made to a Year 13 class in mathematics with calculus. These included observations of lesson implementation, classroom interactions, assessment feedback, and course material. Conversations were held with the classroom teacher that related to teaching philosophy, lesson implementation, learning moments that occurred in class lessons, and feedback from observed lessons. Conversations were also held with selected students.

In the second half of the year, conversations were held with selected students who had studied mathematics at the university level. Most of these students came from secondary schools in the Mathematics Enhancement Project, or schools from the Manukau region of similar demography and socioeconomic status.

This study suggested that students' conceptions of mathematics are generally about knowing procedures, knowing how to do examples, and being good at algebra. The school students interpreted the main challenge in mathematics as "getting the technique right". Attempts by the teacher to introduce a more conceptual teaching strategy were "hijacked" by the students who forced the lesson to follow a more procedural pathway.

The way in which mathematics was institutionalised at university did allow students to glimpse alternative ways of thinking mathematically; ways that challenged them to think about the underlying ideas. However, nearly all of the university students maintained an essentially procedural view of mathematics.

Thus the conclusion is that procedural conceptions of mathematics that students develop during schooling are robust and resistant to change.

As a result of this research, it was concluded that it is essential that students have well-developed algebraic thinking skills, that they have access to a variety of ways of thinking about ideas conceptually, and that they should be exposed to solving problems that require more than a knowledge of procedure.

Pasifika Languages and Mathematics

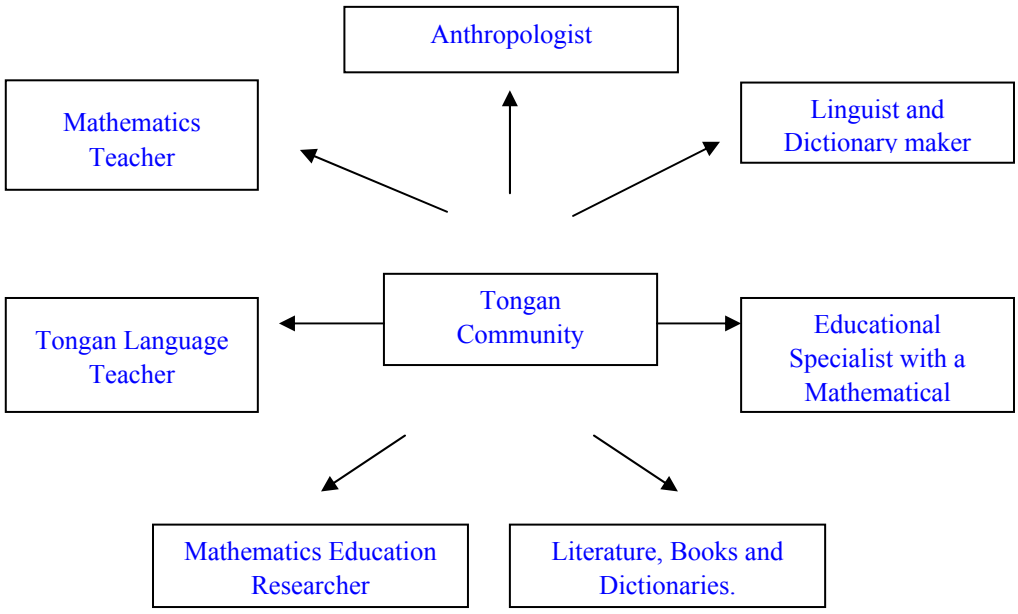
The first year of this study of Pasifika languages and mathematics was also part of the TLRI project directed by Pip Neville-Barton and is detailed in the report of that project. The discussion below is the follow-up in the second year, i.e. 2005.

The second year involved three teachers worked on developing a Samoan/Tongan dictionary of mathematics. This began with an examination of the literature in regard to the relationship between mathematics and language with particular emphasis on Tongan and Samoan literature. Then data on the criteria for word selection that Tongan people have used or preferred were collected from in-depth individual interviews, group discussions, and from relevant materials in the Tongan language.

Interview schedules were used as a reference, to ensure that main areas of interest were canvassed, but most of the interviews were in the form of a conversation rather than a strict adherence to the interview schedule. As well as the interviews, the university researcher participated in an organised discussion group where problems regarding the teaching and learning of mathematics in English to Tongan students were discussed.

Because the study aimed at investigating different criteria that Tongan people from different fields have used or preferred to use when they make new Tongan words, the university researcher selected representatives of the community from the areas shown in Figure 1, as well as exploring existing literature, books that schools have used in the past, books they are currently using, and available dictionaries.

Figure 1 **Community Representatives**



A word list of 50 words was used in the interview as guidelines to create discussions. Because the Tongan mathematics discourse has not been developed, the word list was taken from the topics of numbers, operations, and geometric shapes.

This study remains in development.

Whole-class Discussion about Mathematics

Building on the observation visits from the university researcher and teacher in 2004, Suzanne Kerr made her own observations (13 in total) of two teachers in three Year 13 mathematics (calculus and statistics) classes in two other decile one schools. During each visit, notes were taken which were discussed with the teacher concerned afterwards. Students were surveyed and interviewed about points of interest that arose during the visits. The observations were used to try to identify exactly what was going on in the classroom – what aspects of whole-class discussion promote mathematical thinking, how to develop strategies that would enhance mathematical thinking during discussions, and how to evaluate the amount and quality of mathematical thinking in classroom discussions.

Interviews with students showed that "covering content" can be counter-productive if students do not understand. They identified repeated explanations, fuller explanations, having more time especially on basic concepts, and step-by-step examples as helping their understanding, along with one-on-one tuition.

Lively interactions between teacher and observer continued outside the classroom. Small changes in teaching were made under the impetus of the research, especially in the area of requests for justification of answers to show mathematical thinking. This indicates the way in which small changes of practice can foster greater student participation even within a teacher-centred pedagogical environment. When such engagement was lacking, there was minimal change in teaching practice.

Individual Classroom Studies

The three studies undertaken by individual teachers were not comprehensive enough to be generalised across other classrooms. Two are briefly reported in the Appendices to give an indication of the types of investigations that can be carried out under guidance by teachers who are not engaged in formal research or post-graduate studies.

The studies were as follows.

- The impact of homework on attainment. This study was initiated by the head of mathematics in one of the schools and was a joint effort by the three teachers from that school. A new homework regime was instituted, and its impact measured in homework completion rates and the normal class testing system. The findings—which indicate a strong positive correlation

between completing homework and performing at a high level—were then shared with the students as part of a wider drive to develop students as independent learners. Strikingly, the students who had begun the year with the lowest homework completion rates, and the lowest attainment, ended the year with both homework completion and attainment that was comparable to the best students. This study was not formally written up.

- The factors which cause good students to give up in the final months of school. This study was initiated by the head of mathematics in a school and was eventually adopted school wide, with strategies for supporting students through personal crises being devised (see Appendix 8, p. 38).
- The effect of the teacher pausing in classroom discussion. One teacher instituted an intervention where she deliberately increased the pauses she allowed after asking students questions. This had a dramatic impact (see Appendix 8, p.38).

The Future

Looking to the future, a significant issue for us is the long-term sustainability of any changes that have come about as a result of the project. We can be confident that for some individual teachers, perhaps six of the 27, the opportunity to develop as reflexive practitioners will have continuing benefits for their practice and sense of their own professionalism. These are teachers who have embarked on masters programmes as a result of their work in the Mathematics Enhancement Project, and those who have discovered, in some very concrete way, that the process of trying new approaches in the classroom, reflecting on their impact, and adapting their teaching further in the light of what they find, has had a positive impact on their professional lives.

However, perhaps the more important question relates to the sustainability of the community. This need not necessarily be the current Mathematics Enhancement Project community, but in order for initiatives such as this to have impact that goes beyond improving the situation of a few individuals, there is a need for a community that can facilitate the continuing development of long-term members, induct new members, and which outlasts any individual project or intervention. As previously noted, we have now come to view the development of a sustainable community of reflexive practice to be the most important goal for professional development. We would add that the success of any professional development programme should be measured by the extent to which such a community sustains itself.

This is a tall order, of course, but we believe it is an achievable aim. We doubt that the Mathematics Enhancement Project has achieved this yet, and further research would be needed to identify strategies for promoting such sustainability. But we can make some comments and recommendations on the basis of the work that we have done.

The overall aim of the Mathematics Enhancement Project (that is, beyond this project on professional development research) is not to create a group of teachers who all teach in the same way, but rather to develop a group of teachers who all think deeply about the way they teach, and

have the confidence to develop their own strengths. Any active and energetic community relies on a few key individuals, and those teachers, referred to above, whom we regard as our greatest 'successes' are the ones who would be best placed to take on this role. Thus there is a case for nurturing these teachers, and inducting them into taking on more of a leadership role.

No one project can achieve this alone. It has proven unrealistic to expect a single intervention—even one spanning two years—to single-handedly transform the ways in which teachers engage with each other. However, if other professional development initiatives embraced a community-building model, or if there was a very long-term commitment of some kind, then the cumulative effect could be substantial. Steps such as moving away from prescriptive models of professional development towards those in which teachers are given opportunities to share ideas with each other and develop individual or group responses to external input would begin this process. In this respect, this research reinforces other literature that supports teachers working in clusters of schools.

When working in the low decile context, the systemic issues we have identified cannot be ignored. Our work took place within an existing situation that constrained the possibility for change in these schools. It is important to recognise that no educational intervention alone will ever address the fundamental issue of poverty which is at the root of many of the "problems" associated with low-decile schools. However, we would again argue that professional development which works towards developing communities of professionals who reflect deeply on their practice can have a positive impact. By developing energetic and forward-looking mathematics departments, made up of teachers with a strong sense of their own professionalism and raised morale, more of the schools such as those with which we have been working will come to be seen as exciting and worthwhile places to work, rather than as departments that teachers turn to when they cannot get a job in a "better" school.

A slightly different set of questions relates to the ways in which teachers can be moved to ever-deeper reflection on their own practice. While we were successful in engaging most of the teachers involved in the project in some classroom-based research into their own practice, we were less successful in developing the kind of "critical eye" that leads teachers to question their underlying assumptions. This distinction can be exemplified by the research into the benefits of homework. This research was both rewarding for the teachers concerned, and clearly beneficial for the students. It also developed cyclically, in that the teachers involved brought data along to teachers' meetings and the discussion informed subsequent data collection. Where it falls short of our notion of reflexivity is that the research never moved beyond these teachers' preconceptions of the value of homework; for example, by questioning the nature of the homework that was set. Further research would be needed to explore how this next step might be achieved.

For the immediate future, the university researchers have an ongoing commitment to the Mathematics Enhancement Project as a whole, and there is some funding to continue to support this group of schools and their mathematics teachers. This is planned to restart in Term 3 of 2007, with some funding already allocated.

In addition, two of the university researchers are extending their work with senior mathematics teachers out of the decile one and two environment, and looking at how a group of teachers might be supported towards the specific aim of ongoing mathematics learning for themselves in order to enhance classroom practice.

6. Limitations

Some limitations of the research design must be acknowledged. Being both implementers of a project and researchers into its effectiveness always poses methodological difficulties, and it is possible that there was a tendency for us to see what we hoped to see. Thus, while every effort was taken to ensure maximum objectivity in the collection and analysis of data, we also recognise the impossibility of being truly objective. Although we believe that our close involvement with the project offered insights that may not have been available to a more "detached" observer, this benefit is offset by the potential for findings to conform with expectations.

A similar issue arises in relation to the teachers with whom we worked. Most of the data we collected during the course of the project was with those teachers who were present in meetings, and so it is reasonable to surmise that these teachers too had some investment in the success of the project. If this is the case—as indeed we hope it is—then this may lead to a downplaying of more negative aspects of the project.

In our final interviews we sought to redress this bias by selecting a sample of teachers who captured the diversity of the group, including several who had not remained active participants throughout the two years. However, it is impossible to be sure that our interviewees were not, to some extent, telling us what they thought we wanted to hear, and it is likely that any strong negative reactions will have been toned down.

A further set of limitations relates to the replicability of initiatives such as this. While we believe that our findings point towards an effective model of professional development in low-decile schools, there are a number of caveats. The project is part of a larger project (the Mathematics Enhancement Project) which extends back to 2000 and some (but not all) of the most notable success stories are among the teachers whose active involvement in this larger project goes back furthest. This further reinforces the finding that time is crucial to professional development.

We are convinced that teachers in the decile 1 and 2 schools in which we worked form a particular group, and have some confidence in generalising our results to other teachers in similar schools. However, care is needed when thinking about what we mean by *generalising* in this context. In any programme that depends on the development of a community of practice, the personalities involved are crucial to the form that the community takes. We believe that the success of this project depended on the fact that it built on the particular strengths and interests of the university team whilst remaining responsive to the particular needs and interests of the teachers involved. Any attempt at replicating the project by following the steps that we took could be missing these vital characteristics. Thus what is generalisable in this case is not a specific set of actions but rather an overarching approach.

7. Capacity Building

Capacity building took place on several levels: amongst the university researchers, amongst the teachers who engaged in tertiary study, and amongst the other teachers.

Amongst the university researchers, there were several ways in which this project built capacity for further research in mathematics education.

The interest and involvement of experienced overseas researchers in a project with a New Zealand context has been an extremely valuable feature of the project. Not only did they reflect what they saw in New Zealand with their own experiences in a variety of countries (Brazil, France, Australia, England), but also they brought new methodological ideas (regarding culture from Brazil and regarding classroom interaction from France) that are not common in this type of research in English-speaking countries. The whole team benefited from this input and now has access to these ideas (and these collaborations) for the future.

The principal investigators were both experienced researchers but one in particular did not have a lot of experience with large projects. This work built their managerial research experience.

The other four researchers are at an early stage in their research careers: two with doctorates and two with masters degrees (one of whom has gone on to a doctorate). The project was a wonderful environment for them to undertake research as part of a multi-faceted team of experienced researchers and classroom practitioners.

Several publications have emanated from the project, and three of those were first-time publications for some of the authors. They were all accepted by a peer review process.

The four teachers who engaged in tertiary study through Ministry of Education Study Awards or Royal Society of New Zealand Teacher Fellowships while participating in this project have probably had the most benefit. These were teachers who had not previously thought about research as part of their professional lives, who had not been motivated to continue their own study of any kind. Through the collaborations of the project they perceived opportunities for research and study and decided that it was time to move forward.

> I did . . . two papers at university this year . . . I'm pretty sure I wouldn't have [if it weren't for the project]. . . . Now I'm thinking about actually finishing off a . . . PG Dip Sci or . . . working towards a masters, which I would never have considered before. And I think those things have brought a lot into my teaching too. (MEP teacher)

All courses undertaken (this included formal papers as well as special topic studies related directly to the project) have been completed successfully, and one teacher has completed a

Postgraduate Diploma in Science in this time. These teachers are all still in the classroom and have returned renewed and with new perspectives on practice.

The other teachers have benefited in a range of ways. The participation in a community of practice was the aspect that received the most comments, and the teachers' enthusiasm for contact with each other was increasingly visible in the meetings as the project continued.

> As I said the other day, it's really good to meet all the other people around the same field. And now I feel – Yes! If I have any difficulties, any issues, I can always have someone I know who I can discuss with. That's a huge difference. It's built a community. (MEP teacher)

Those six teachers who participated in their own research project to a point where they achieved results have this experience to build on, although they may not independently repeat such a study. All the teachers learned new mathematics through the project, and some conventional workshops had the usual output of new ideas tried in classes, although the lasting effect may not have been significant.

> MEP becomes very relevant for me as a person, as a teacher. . . . We usually don't get time to see a bigger picture or some deeper things which actually should be keeping our overall balance in the right place. We are missing out on the mathematical world of that nature. (MEP teacher)

We were left with no doubt that this has been a positive involvement for the teacher-researchers, and compares extremely favourably with other professional development experiences. In the interviews teachers commented on this project in relation to other recent professional development initiatives, and in every case the comparison was favourable. The open nature of this project, and its community of teachers, were given as reasons.

8. Research Outputs and Dissemination

This project has given rise to several conference presentations and publications.

The major presentation was to the 15th ICMI [International Commission on Mathematical Instruction] Study on Teacher Development, Brazil in May 2005. Hannah Bartholomew presented a paper written by the whole team to this international invitation-only study. Papers are pre-prepared, refereed, and on the basis of this process invitations are made. The ICMI Study will result in a book that is written from the working of the conference rather than as a collection of presentations. This is in process. The focus of our paper was the emotional investments that teachers have in particular roles, and this was used to theorise the threatening nature of making changes that may challenge or undermine these roles. This struck a particular chord with a number of the participants. We have also been invited to contribute to a chapter which develops these ideas to a volume on teaching mathematics in challenging circumstances, edited by Robyn Zevenbergen and Mike Askew. See:

> Bartholomew, H., Barton, B., Kensington-Miller, B., & Paterson, J. (2005, May). *Mathematics teacher development in low socio-economic areas.* Paper presented at the 15th ICMI Study on Teacher Development, Brazil.

Shortly after the ICMI Study, the international Mathematics Education and Society conference was held in Gold Coast, Australia (July, 2005). This four-yearly conference is the main forum for issues of equity, politics and culture in mathematics education. We wrote a follow-up paper to the ICMI Study that was presented by Hannah Bartholomew and Barbara Kensington-Miller. This paper uses the metaphor of a dragon to represent teacher fears of exposure, and theorises the critical role of these fears in teacher development. It links the overcoming of these fears to the development of communities of practice. The paper was published in the refereed proceedings:

> Barton, B., Paterson, J., Kensington-Miller, B., & Bartholomew, H. (2005, July). *Dodging the dragon: Strategies for mathematics professional development in low socio-economic areas.* Paper presented at the 3rd Mathematics Education and Society conference, Australia.

Later in 2005, the fifth DELTA conference was held in Australia. This is the southern hemisphere forum for tertiary mathematics education. Judy Paterson presented her work on mathematics as professional development, which was published in the proceedings:

Paterson, J. (2005). Flicking the switch: Using mathematics to reconnect mathematics teachers with their learner selves. In *Proceedings of the fifth southern hemisphere symposium on undergraduate mathematics and statistics teaching and learning*, (pp. 103–116). Fraser Island, Queensland: Kingfisher Delta:05.

The annual mathematics education conference in the region is MERGA (Mathematics Education Research Group Australasia). In 2004, 2005 and 2006, Barbara Kensington-Miller presented papers on her mentoring project, and in 2005 Viliami Latu did the same on the Pasifika study. Papers at MERGA undergo a blind peer review process and are published in refereed proceedings:

Kensington-Miller, B. (2004). Professional development of mathematics teachers in low socio-economic secondary schools in New Zealand. In D. Putt, R. Faragher, & M. McLean (Eds.), *Mathematics education for the third millennium: Towards 2010. Proceedings of the 27th annual conference of the Mathematics Education Research Group Australasia (MERGA–27) Vol. 2* (pp. 320–327). Townsville: MERGA.

Kensington-Miller, B. (2005). Mentoring mathematics teachers in low socio-economic schools in New Zealand. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, et al. (Eds.), *Building connections: Research, theory and practice. Proceedings of the 28th annual conference of the Mathematics Education Research Group Australasia, (MERGA–28) Vol. 2* (pp. 459–466). Melbourne: MERGA.

Kensington-Miller, B. (2006). The development of a community of practice and its connection with mentoring in low socio-economic secondary schools in New Zealand. In P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds.). *Identities, cultures and learning spaces. Proceedings of the 29th annual conference of the Mathematics Education Research Group Australasia (MERGA–29) Vol. 2* (pp. 320–327). Canberra: MERGA.

Latu, V. (2005). Language factors that affect mathematics teaching and learning of Pasifika students. In P. Clarkson, A. Downton, D. Gronn, M. Horne, & A. McDonough (Eds.), *Building connections: Research, theory and practice. Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia (MERGA–28)* (pp. 483–490). Melbourne: MERGA

The project has also given rise to theses within the University of Auckland's Department of Mathematics as part of masters studies. These are:

Kerr, S. (2005). *Whole class discussion about mathematics*. Unpublished masters thesis, University of Auckland.

Latu, V. (2004). *Language factors that affect mathematics teaching and learning of Pasifika students.* Unpublished masters thesis, University of Auckland.

Nathan, G. (2005). *Students' conceptions of calculus: A study of a group of year 13 and first year university mathematics students.* Unpublished masters thesis, University of Auckland.

The project was also presented as a seminar to the mathematics education group at Kings College, London, by Hannah Bartholomew.

Hannah Bartholomew. (2006, April). *Emotion and mathematics professional development in a low socio-economic area in New Zealand.* Paper presented at Kings College, London.

The Mathematics Education Unit in the University of Auckland holds a seminar series that provides a forum for reports on research in mathematics education. This has been used as a forum for disseminating information on this project. Presentations made are as follows:

- June 24, 2005, Viliami Latu, "Language Factors That Affect the Mathematics Teaching and Learning of Pasifika Students"
- October 21, 2005, Judy Paterson, "Continued Investigation Within the MEP Project Into Using Mathematical Stimuli to Encourage Teacher Talk and Reflection About Learning and Teaching. A Discussion of Findings From Initial Analysis and Current Development of an Analytical Tool"
- September 25, 2005, Garry Nathan, "Students' Conceptions of Calculus: A Study of a Group of Year 13 and First Year University Mathematics Students"
- June 23, 2006, Barbara Kensington-Miller, "The Development of a Community of Practice and its Connection With Mentoring in Low Socioeconomic Secondary Schools in New Zealand"
- August 4, 2006, Judy Paterson, "A search for Underlying Mechanisms in a Model of Teacher Change"
- August 17, 2006, Suzanne Kerr, "Whole-Class Discussion About Mathematics"
- August 25, 2006, Hannah Bartholomew, "Emotion and Mathematics Teaching in a Low Socioeconomic Area".

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Barton, B., Paterson, J., Kensington-Miller, B., & Bartholomew, H. (2005, July). *Dodging the dragon: Strategies for mathematics professional development in low socio-economic areas.* Paper presented at the 3rd Mathematics Education and Society conference, Australia.

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Appendices

Appendix A: Professional Development Through Mathematics

The most up-to-date published report of the progress in this study may be found in the 2005 DELTA paper (see reference below). A PhD thesis on the study is currently being written and will be available in 2007.

Paterson, J. (2005). Flicking the Switch: Using Mathematics to Reconnect Mathematics Teachers with their Learner Selves. In *Proceedings of the fifth southern hemisphere symposium on undergraduate mathematics and statistics teaching and learning* (pp. 103–116). Fraser Island, Queensland: Kingfisher Delta:05

Appendix B: Collegial Mentoring Between Mathematics Teachers

A full report of the progress in this study may be found in the 2006 MERGA paper. A PhD thesis on the study is currently being written and will be available in 2007.

Kensington-Miller, B. (2006). The development of a community of practice and its connection with mentoring in low socio-economic secondary schools in New Zealand. In P. Grootenboer, R. Zevenbergen, & M. Chinnappan (Eds.), *Identities, cultures and learning spaces. Proceedings of the 29th annual conference of the Mathematics Education Research Group Australasia (MERGA–29) Vol. 2* (pp 320–327). Canberra: MERGA.

Appendix C: Questions of Identity as Mathematics Learners

As has been noted elsewhere, the original conception of the teacher research within this project – and the model followed for the first year – involved university researchers working with a small group of teachers on research projects relating to a particular theme. One of these themes was student identity, and Hannah Bartholomew worked with a group of 5 teachers on research projects linked to this idea. For a number of reasons, this model had disappointing results; we will focus here on issues relating to the identity theme, rather than those which relate more generally to the wider project.

With the benefit of hindsight, we suspect that students' identity is rather too esoteric a topic for research if this kind, and defining appropriate research questions with teachers proved difficult. Furthermore, the fact that it was so clearly about students rendered it an ineffective tool for engaging teachers in deep reflection about their own practice. Indeed, at times it seemed that the research was reinforcing existing deficit views of students.

This is not to say that the time invested in working with teachers on this topic was wasted. Hannah Bartholomew visited the classrooms of the teachers with whom she was working, and the discussions she had with teachers were fruitful. One approach that steered discussions towards a focus on teachers' own practice while remaining true to the identity theme, was to talk about 'ownership'. A number of very productive visits to teachers involved discussion of which aspects of the lesson had enabled students to claim a sense of ownership of the material covered, and how this could be promoted further. While being useful, however, we never really felt that these times were really successful in engaging teachers in research.

In the second year of the project we adopted a different approach to teacher research. An important feature of this model was that the research process rather than particular research topics were foregrounded, and teachers were invited to select a topic that was of interest to them. However, a number of teachers continued to pursue individual research that related to identity. In one case, an interest in the ways some students appear to 'give up' in Year 13 led one of the teachers to speak to students about this, to mentor them through the year, and to involve the Year 13 dean and school counsellor in an initiative which aimed to prevent this from happening.

Despite the variable success of this work with teachers on students' identity, the university team found working with the notion of teachers' identity to be a very useful theoretical tool.

In an important sense, a key aim of the project was to make a particular kind of professional identity available to these teachers. As we have reported elsewhere, many of them are

comparatively isolated in their schools, and are locked into a daily routine in which 'coping' is the best that is hoped for. Our project, with its focus on developing a Community of (reflexive) Practice, rather than providing answers or modelling 'best practice', aimed to create the space for teachers' professional identities to develop.

In explicitly rejecting the notion that we have 'answers', and instead seeking to nurture teachers' capacities to find their own answers, we are embracing the fact that different teachers will develop in different directions. A teacher's professional identity is part of a complex web of biography, beliefs and values, and current situation.

Particularly significant for the teachers in this study was the kind of school in which they were working. We quickly identified a variety of systemic features of these environments that had a significant practical impact on their professional lives. What was less immediately apparent, but something we became increasingly aware of, was the profound effect on the psyches of these teachers of working in decile 1 and 2 schools. We would argue that developing the professional identities of teachers demands attention to – and sensitivity towards – the psychological at least as much as to the practical.

Interviews conducted with teachers at the end of the study highlighted both the tremendous diversity across the group, and a number of common themes. In order to illustrate our approach we will briefly review two contrasting responses to teaching in a low decile school. While the typologies described below capture something real in our data, it should be noted that most teachers do not fit neatly into one or other category. (n.b. We are still developing these ideas; we have previewed them in two conference papers, and they will be expanded in a book chapter that we have been invited to write).

There was one group of teachers who, when interviewed, conveyed a powerful sense of vocation about working in "disadvantaged areas", or with "under-privileged kids". These teachers were clear that they would not want to work in a high decile school, and that it was in schools such as those where they worked that they could "make a real difference". In meetings, teachers in this group frequently spoke up in defence of their highest attaining students, whom were felt to be "much better than a student from [a decile 10 school] with the same grades – because you know they've had to really fight for it". This sense of 'making a difference' was clearly a source of considerable professional pride for these teachers.

A second group of teachers spoke of the undesirability of the school in which they worked. These were teachers who would prefer to teach in a higher decile school, and seemed to have internalised a pecking order of schools which was closely correlated with decile rating. Many of the teachers in this category were immigrants from non-English speaking countries, and felt themselves to be (and probably were) disadvantaged when applying for jobs in "better" schools. These teachers more frequently cited the practical constraints they faced, and saw these as insurmountable barriers which prevented them from doing their job as well as they would like. A sense of professional pride was more tenuous among teachers in this group, but appeared to be founded on a their ability to 'cope in the face of adversity'.

While very different responses, the two vignettes above have certain things in common. In seeking to unpick these differences and similarities we have found it useful to think in terms of teachers' defensive investments in particular subject positions. If we accept that everyone has a range of defensive strategies in order to protect themselves from vulnerabilities, then both sets of responses above can be understood in this way. Central to both type of teacher response is a conviction that their school is fundamentally different from 'other' schools – in one case the teachers are fiercely protective of their own school, and in the other it is regarded as a problem, but the sense of its difference is common to both sets of responses. While it is clearly true that these schools are different, it is the passion and frequency with which this difference is stated that marks it out for us as a key feature of these teachers' defence mechanisms: it is a story teachers repeat, to themselves and others, in order to strengthen their sense of professional pride. All of the teachers with whom we were working on this project have to find ways of coping with the fact that the schools in which they work are commonly regarded unfavourably. They do not have the profile of higher decile schools, and public awareness of them is more likely to centre on their 'problems' than on their 'successes'. Whereas teachers in schools that are regarded as successful are likely to receive considerable external affirmation of the work they are doing – for example in relation to well-above average attainment of students – these teachers are more likely to have to find their affirmation internally.

For us, the strength of this model is that it enables us to understand some of the resistance we encountered from teachers in terms of the extreme risk to their professional identities that change entailed. A response that was often heard when more 'open' teaching strategies were discussed was "but we couldn't do that with our kids". A recognition of the strong investment that these teachers have in the difference of their schools from others makes it possible to understand responses such as this as reflecting the risk not just of failing, but also of succeeding – because discovering that they could 'do that with their kids' would challenge the defensive structure on which their sense of professionalism rests

Insights such as this bring with them no easy answers. For us they again reinforce the need for time to develop a community of practice which is a safe space for teachers to both experiment and to engage with these issues. They also highlight the ways in which seemingly innocuous activities can tap into deep-seated vulnerabilities. But perhaps the most significant issue that is raised by the responses of these teachers is the hierarchy among schools…

Appendix D: Using the Didactic Contract in Mathematics

The intention of this theme was to use the concept of didactic contract, as elaborated by the French theorist Brousseau, as a tool for reflection on classroom activities. A didactic contract is an implicit or explicit agreement between teachers and student(s) about an aspect of the mathematical learning process. The theme was adopted by a group of three teachers in the study.

The study method was to use the teacher meetings to concentrate on one aspect of the classroom and attempt, in discussion, to frame it using the idea of didactic contract. The teachers' understanding of the didactic contract(s) involved were then tested in the classroom through observations made by the university researcher in the classroom. These observations were then debated for their meaning in terms of the contract in an email dialogue between the university researcher and the individual teacher. A record of the observation and dialogue was kept. A sample of the interaction is given below: T indicates teacher, U indicates university researcher, S1, S2 refer to students.

T. Lesson Records **29.04.04 Year 13 MAC 17 students**
Good-natured greeting of me as well as interactions with T before lesson started. U fiddles with sheet for OHP screen. T discusses trip to university, arrangements and requirements. They are interested in who else is going. One comment from S1 was about "Oh, you mean the dumb schools" then "Decile 1". U asks how many have been to university before (three – girls + 1??). Also how many use email: all claim to at the school. Then lesson proper starts. T goes over some homework questions, eliciting feedback for steps in their solution. Answers (mostly correct) are called out, checked out by T, and written down. U adds some comments at the end. Then U gives a session on swings and sine curves, finding slopes and hence the derivative, namely cosine, then the second derivative. T extends this with a distance → velocity → acceleration diagram, and U comes back in with a discussion of the equations of motion using the diving gannet example. He asks the class to convert m/s to kph. When asking for the result of the calculation, S2 attempts to get everyone to chorus it together so that no-one will feel shame if they get it wrong. In another incident, when U was explaining the second derivative notation and used d^2y/dx^2, then T came in and showed a "teacher's trick" to help remember what it means.

U: I was surprised by the "Dumb schools" comment. I wonder whether you get this sort of comment very often? My inclination is to have a frank discussion about it. I wonder whether this attitude extends to themselves "I am dumb because I'm at a dumb school"?? How do we get round that one? Does this actualise into classroom mathematical behaviour. For example, when they come up against something difficult, they do they more readily give up because they think they are dumb, or possibly think we think they are dumb and so won't expect them to work it out?

T: I did have a talk to them about it. We discussed what 'dumb' meant and how that might relate to the school, the district, themselves, etc. They mentioned poverty in their families and the district and I pointed out that that was what Decile 1 meant. When we discussed how that might relate to them, they said that although the school had good facilities etc (they are very aware o f this and it is a source of pride to them) that the school 'didn't get results like other schools'. We then discussed why that might be, are they 'dumb', and also the perennial topic of homework!! An interesting comment was made by one of the brighter girls S3 that when you can't do something, you don't feel like carrying on with homework. We then had a long harangue by me using the analogy of practising for a sport. (One of the big problems is that these students want things to be easy, they certainly don't want to put the time and effort in (or don't ***have*** *the time), maths is bottom of the heap. We discussed this at the MEP day on the 4th. Something else that maths comes behind (that we discussed as a contract) is work. These students do huge amounts of paid work. Last year one student couldn't come to a practice UB exam on a Saturday because she had to work! When I went bonkers that she was still working at that point in the year (the weekend in the middle of UB exams!) she defended herself by saying that she wasn't actually working on exam day!) I try to encourage the students to do their maths homework FIRST before they get tired. Any other suggestions?)*

U: I'm interested by two things here. One the idea that "they want things to be easy". I have a feeling that it is a bit more complicated than this – and the observations of the next lesson seem to contradict this idea. They want to be able to do it, sure, but they also like it to be new and challenging? If I think of my own learning, I enjoyed being able to do more difficult things, being able to do easy things did nothing for me.

The second question is this one of work. We should ask them about their priorities. Do they really need the money – or do they prefer work to homework?

The second incident that made me think was S2 trying to get everyone to chorus the answer together so that he (and others?) would not be shamed if they got the right answer. I could phrase this as a Contract:

Contract 4: It is bad to get a wrong answer.

I read somewhere of a classroom where wrong answers were more valued that right ones because they created opportunities for everyone to learn.

T: I enjoyed the lesson and continued in the next lesson with the terms and notation of speed/velocity/acceleration.

U: So did I enjoy it, very much, especially the way you and I interacted, inserting new points, picking up from each other. I wonder whether we can model mathematical thinking and mathematical discussion between us??

T: One thing the students say they find difficult is 'knowing what to do when'. I try to give them heaps of vocab, constantly go over it, do problems in several ways, ask them what clues there are from the context, what the words might mean (eg'changing direction' means ...)

The technique was successful in promoting critical discussion directly in one case, and indirectly for the other two teachers.

In the directly successful case, the situation under discussion was whole class discussion, that is, when the teacher was explaining a mathematical concept to the whole class at once in a way that involved blackboard elaboration, questions, answers, and some student participation at the blackboard. Contracts were identified, particularly surrounding the way questions were to be answered and the ways that students could interact with each other to arrive at answers. Some contracts concerning student/student interaction were also noted. This proved to be an excellent basis for subsequent email discussion of the classroom incidents and critical reflection of the teacher's practice.

This case led to this particular teacher following through with whole class discussion as a research project in the second year under the terms of a Study Award.

The other two teachers were both concerned about students taking responsibility for learning, particularly with respect to homework. While it did not lead to useful critical discussion in the first year, these two teachers with their HoD were a team that took the theme of homework as their self-directed study in the second year. It is difficult to tell whetherthe framing as a didactic contract helped them to think about the parameters of the second study.

Appendix E: Conceptions of Calculus

This study explored the nature of conceptions of mathematics students develop as they work with mathematics in increasingly complex ways. For most students calculus is the first time they meet the complexities of mathematics and its ability to help them make sense of their world. It enables students to think about mathematics in ways that begin to give it some structure – some internal consistency. The question at the centre of this investigation was; 'What is the conception of calculus for Year 13 Secondary School Mathematics with Calculus' students compared with first year undergraduate university students who are studying calculus?' Exploring this question involved an examination of how conceptions are structured, and how they influence the ways in which students engage in mathematics.

Over an approximate school year period, weekly visits were made to a secondary school involved in the Mathematics Enhancement Project. Observations were made of a Year 13 Mathematics with Calculus class with the co-operation of the Principal, and Head of Department. These included observations of lesson implementation, classroom interactions, assessment feedback, and course material. Conversations were held with the classroom teacher that related to teaching philosophy, lesson implementation, learning moments that occurred in class lessons, and feedback from observed lessons. Conversations were also held with selected students and these focused on their concepts of mathematical ideas, learning mathematics – in particular what is important in coming to know mathematics, and their role in the mathematics classroom.

In the second Semester of the University academic year, conversations were held with selected students who had studied mathematics at the university level. The students selected had studied at least two mathematics courses, one of which they all had taken. The conversations focused on establishing these student's conceptions of mathematics. Of note is that most of these students came from secondary schools in the MEP project, or schools from the Manukau region of similar demography and socio-economic status.

This study suggested that student's conception of mathematics is generally one of knowing procedures, knowing how to do examples, and being good at algebra. The school students interpreted the main challenge in mathematics as 'getting the technique right'. This is in contrast to the current wisdom of mathematics education where importance is given to understanding, and being able to make a connected sense of mathematical ideas. Attempts by the teacher to introduce a more conceptual teaching strategy were 'hijacked' by the students who forced the lesson to follow a more procedural pathway.

The way in which mathematics was institutionalised at university did allow students to glimpse alternative ways of thinking mathematically, ways that didn't only focus on techniques and rules,

but challenged them to think about the ideas that underlie them and the structure of these ideas. Important to creating opportunity for a shift to a more conceptual view of mathematics was the teaching process, and the use of examples that illustrated the usefulness of mathematics. However, nearly all of the university students still maintained an essentially procedural view of mathematics.

This study suggests that procedural conceptions of mathematics that students develop and identify with throughout their schooling are robust and resistant to change. As noted by the case of the university students, change to a more conception view can occur, but when it does it occurs very slowly and in little steps. A maturing connected view of mathematics was only evidenced in one student who was taking a second year mathematics course.

As a result of my research and subsequent teach at tertiary level, I now think that it is essential that students have well developed algebraic thinking skills. They should also have access to a variety of ways of thinking about ideas conceptually, for example through symbols, imagery, and the use of formal mathematical statements. Students should be exposed to solving problems that require more than a knowledge of procedure, problems that call on understanding the big ideas that may underlie them. Thus, it is my opinion that it is not sufficient for teachers of Year 13 secondary school mathematics to focus their teaching only on the Achievement levels of the NCEA assessment requirements. Teaching should also include a substantial amount of material pertinent to the Merit and Excellence levels, as these illustrate the usefulness of mathematics, the logical structures that underlie them, and invites students to engage mathematics as 'mathematicians'. At the university level, students must be exposed to robust images of ideas, images that will lead from sound intuitive bases of mathematical ideas to more formalized ones. Students should also be engaged in developing their personal mathematical discourse, and learn through interactions what counts as mathematical argument and explanation.

Appendix F: Pasifika Languages and Mathematics

A report of this study may be found in the 2006 MERGA paper. A Masters thesis on the study has been completed.

Latu, V. F. (2005). Language factors that affect mathematics teaching and learning of Pasifika students. In P. Clarkson, A. Downton, D. Gronn, M. Horne, & A. McDonough (Eds.), Building connections: Research, theory and practice. Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia, (MERGA–28) (pp. 483–490). Melbourne: MERGA.

Latu, V. (2004). *Language Factors that Affect Mathematics Teaching and Learning of Pasifika Students*. Unpublished masters thesis, University of Auckland.

Appendix G: Whole-class Discussion About Mathematics

Teachers working in low-decile, Polynesian schools face particular difficulties. Students are process orientated – they want notes, a method and practice - and prefer one-on-one explanations rather than whole class teaching. Students often have English as a second language and subtle use of simple words in mathematics is difficult for many. Discussion in the mathematics classroom is an important tool for successful teaching and learning, particularly in schools where the students are already at high risk of education failure as a consequence of their socio-economic background. During my involvement in MEP, I became interested in the idea of Enhancing Whole Class Discussion as a topic for research.

In 2004 University of Auckland researcher Bill Barton made seven visits to my Year 13 Mathematics (Calculus) class. In 2005 I made my own observations (thirteen in total) of two teachers in three Year 13 mathematics (Calculus and Statistics) classes in two other decile one schools. During each visit, notes were taken which were discussed with the teacher concerned afterwards. Students were surveyed and interviewed about points of interest that arose during the visits. We used the observations to try to identify exactly what was going on in the classroom - what aspects of whole class discussion promote mathematical thinking, how to develop strategies that would enhance mathematical thinking during discussions and how to evaluate the amount and quality of mathematical thinking in classroom discussions.

Interviews with students show that 'covering content' can be counter-productive if students don't understand. They identified repeated explanations, fuller explanations, more time especially on basic concepts, and step-by-step examples as helping understanding, along with one-on-one tuition.

Finding a balance between a classroom environment that is open to student ideas and one whose purpose is to learn specific mathematical content is acute in secondary schools with concerns about giving students success, covering the curriculum, and fulfilling examination requirements. The best focus is to build on existing knowledge, draw parallels, and encourage students to actively make connections and extend their current knowledge.

Lively interactions between teacher and observer continued outside the classroom. Small changes in teaching were made under the impetus of the research, especially in the area of requests for justification of answers to show mathematical thinking. This indicates the way in which small changes of practice can foster greater participation even within a generally transmissive environment. When such engagement was lacking, there was minimal change in teaching practice.

Some research claims improvements in student performance simply by giving more time to answer teacher questions. I did not measure wait time, or even really consider it but it could be an area for further research in these classrooms. I think decile one students would benefit from the introduction of a 'slowing down' mechanism to the classroom.

Good teacher questioning with increased wait time, expectations of student justification, and 'slowing down' are simple changes to make which could ensure better outcomes for the students in the schools studied. "Further research on classroom discourse needs to provide more evidence of the practices of successful teachers of diverse students." (White 2003)

White, D. Y. (2003). Promoting productive mathematical classroom discourse with diverse students. *Journal of Mathematical Behavior*, *22* (1), 37–53.

Appendix H: Two Teacher Studies

The following are brief accounts of two of the studies carried out by individual teachers in the second year of the project.

Study A Good students "giving up"

One of the teachers who had worked on the 'identity' research study in the first year of the project, continued to develop this theme in the second year, as an independent study. His particular interest was in those students who appeared to 'give up' as they neared the end of school: students who had done well in earlier years and who were expected to get good marks in Year 13, but who appeared – in the final few months – to either cave under the pressure, or to become distracted with personal issues and lose interest in school work.

Having seen this happen to a number of students in previous years, he was interested in researching possible interventions that might help keep such students focused. He identified those whom he considered to be 'at risk' and spoke to them early in the school year about the pressures of Year 13, and his hope that they would be able to cope with these and succeed. Over the year he built a relationship with these students that enabled them to talk to him about difficulties that they were experiencing with schoolwork and in their personal lives, and at the end of the year they were all studying hard for their upcoming exams.

In addition to this intervention on an individual level, the teacher discussed this work with the dean, principal and school counsellor, and was instrumental in initiating a whole school mentoring policy.

The value of this study is not in its validity as a piece of research, but rather in the fact that the process of doing this research facilitated the development of a very different kind of relationship between the teacher and his students. This was not only potentially life-changing for the students concerned, but also extremely affirming for the teacher himself. In an interview conducted towards the end of the project, it emerged that he had had an extremely rich and varied career, moving from India as a young man and working in Fiji, Samoa and on a United Nations Development Project in the Cook Islands, before arriving in New Zealand. Reflecting on his career, his words are infused with a love of mathematics and a real compassion for the students he has worked with, and he spoke with great emotion about several students for whom he had made a real difference in the past.

Yet despite these experiences, and evident successes, when he first became involved in the MEP he appeared worn down, and unsure of what he had to offer in his current position as head of maths at one of the MEP schools. We would argue that engagement in research – and in

particular, in an individually defined research project during the second year of the study – was an energising experience that allowed this teacher to re-connect with his own passions and strengths.

Study B Teacher pausing

Another teacher, after listening to a colleague talking about whole class discussions, picked up an idea for her own classroom study. She decided to try and initiate more student conversations in her mathematics classroom by pausing more often when she was teaching.

She then initiated an intervention by changing her own behaviour. When she was explaining something, she would:

- pause after she had said something important;
- physically step away from the blackboard and look at it;
- look at the students without necessarily asking them anything; and
- actively look for cues given by the student that they wanted to say something.

This did not mean that she was talking less, or making less of the lesson teacher-directed. In this way she hoped to:

- engage the students in more ownership of the topic they were learning about;
- encourage students to reflect on what they had picked up from the lesson;
- encourage students to explain to others what they had learnt;
- model refection and exchange of ideas to the students; and
- get students to verbalise what they were learning.

The result of this intervention was that:

- there were more student conversations in the classroom;
- she found herself to be more content that the students were talking;
- she was more focused on whether the students were engaged in the lesson;
- the students helped each other more; and
- the lessons were more relaxed and better paced to the students needs.

This study was a classic piece of informal action research carried out by an individual teacher in their own class and leading to improved practice in a short space of time. Notable features were the simplicity of the intervention and the significant nature of the result.

This study was reported back to the entire study community at a later meeting and generated considerable discussion and interest. Several teachers appeared motivated to try this intervention for themselves. (There was insufficient time to gather information on whether this in fact happened or the effects, if any).