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RESEARCH INITIATIVE
NĀU I WHATU TE KĀKAHU, HE TĀNIKO TAKU

Learning to “friendly argue” in a community of mathematical inquiry

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Introduction

This project explored the sorts of culturally responsive pedagogy teachers can engage in to optimise equitable access for students to proficient forms of mathematical talk and activity. The project sought to further our knowledge of the effects on student achievement and mathematical disposition when a specific focus is placed on building a classroom culture of mathematical inquiry and argumentation.

Key findings

- The teachers were able to create more opportunities for collaborative group activities focused on mathematical inquiry.
- The students' role in inquiry mathematics learning activities required a dramatic change from passive receiver to active learner.
- Students developed new ways of thinking about mathematics and their relationship with mathematics, and came to view their roles differently.
- Focusing on the cultural, social and mathematical wellbeing of all the students resulted in positive learning outcomes for Pasifika students.

Major implications

- Professional development for teachers to create learning environments that help all students to learn mathematics offers a valuable way to address underachievement among Pasifika and Maori students.
- Changing the ways students engage with mathematics through regular participation in collaborative group tasks can result in major improvements in student learning outcomes.
- When assessing learning we need to look beyond improvement in mathematical scores to how students engage with mathematics and the relational aspects with their peers.

The research

The overall aim of the project was to examine and explore the sorts of pedagogical practices that optimise equitable access for all students to increasingly sophisticated forms of mathematical discourse and mathematical practices. The project's specific objectives were to:

- introduce teachers to a communication and participation framework (CPF) tool (Hunter, 2007) used to support development of students' participation and communication patterns and through which the numeracy achievement levels and positive mathematical dispositions for diverse learners are enhanced.
- strengthen understanding of how *generative* change can be supported using a CPF tool designed to promote reflecting-in-action skills that directly address student mathematical behaviour, thinking and disposition.

Research questions

The four research questions were:

- How do teachers adapt and use a purposely designed communication and participation framework to structure student engagement in collective reasoned discourse?
- What different pedagogical strategies and adaptations do teachers make to position diverse students to access mathematical discourse equitably?

- How are changes in students' ways of participating in their mathematical learning influencing their mathematical proficiency and identity?
- How do students view their roles and responsibilities in the collective discourse as they gain increased agency in the classroom mathematical discourse?

Methodology

The study used a design research approach (Design Based Research Collective, 2003). A collaborative professional learning cluster, comprising two researchers and four teachers (two from each of the two schools), met at 3-monthly intervals. Documentation of the processes and activities of the cluster group included audio recordings of group discussions, and written artefacts (including each teacher's pathway in implementing and using the CPF, and teachers' reflections on the effectiveness of the pedagogical strategies).

Data collection in the classrooms included regular (3-monthly) video-captured records of mathematics classroom lesson events. Lesson events included the first section of each lesson, when the teachers structured the interaction patterns and mathematical activity; the teacher interactions with students working in small problem-solving groups and/or student–student interactions within the small groups; and the concluding group session. Interviews with selected students were undertaken immediately after the video-recorded lessons. The students were offered the opportunity to review photo records and to discuss and describe specific sections of mathematical talk and activity from their perspective.

Data on student numeracy and mathematical levels were also collected (GLOSS and PAT norm-referenced tests). The video records, student interviews and achievement results provided a platform for the cluster group members to reflect on and adapt the classroom communication and participation patterns in ways aimed at supporting increased participation and empowerment of diverse learners. Particular attention was paid to how the teachers drew on students' ethnic socialisation (Macfarlane, 2004) to enable a social and cultural fit.

Findings

The CPF tool proved to be an effective approach to supporting both professional learning and classroom activity. The teachers used the CPF to support their efforts to gradually scaffold the students to make increasingly proficient mathematical explanations, representations, justifications and generalisations—key practices that facilitate mathematical argumentation and the construction of rich and connected mathematical knowledge.

Drawing on the CPF, the teachers put an initial focus on a set of communicative and participatory actions that required the students to:

- provide a mathematical explanation using the context of the problem, not just the numbers
- provide mathematical reasons/justification for their thinking, rather than just a description of their thinking
- as a group, explore two or more ways to explain a strategy solution
- analyse explanations and construct ways to revise, extend and elaborate on sections others might not understand
- predict questions that could be asked by peers, and prepare mathematical responses
- ask questions that clarify an explanation (What do you mean by? What did you do in that bit? Can you show us what you mean by...? Can you draw a picture of what you are thinking?)
- agree on the construction of one or more solution strategies that all members can understand and explain
- work together to check, explain and re-explain in different ways the group explanation.

For many students, learning and becoming confident to construct, present and question mathematical explanations was a lengthy process, which required teachers to continually press students to provide conceptual mathematical explanations. To achieve this, teachers needed to gradually build on and extend their expectations for the students to engage in justification and mathematical argumentation.

Among the cluster group, the CPF served as a reflective tool for the teachers to analyse and plan culturally responsive ways to institute additional communicative and participatory actions. This included requiring the students to:

- indicate agreement or disagreement (with mathematical reasons) for part of an explanation or a whole explanation
- justify an explanation using language such as, “I know $3 + 4 = 7$ because $3 + 3 = 6$ and 1 more is 7”
- use exploratory language such as, “so”, “if”, “then” and “because” to justify and validate an explanation
- use questions that lead to justification (e.g., How do you know it works?, Can you convince us, Why would that tell you to, Why does that work like that?, Are you sure it’s...?, So what happens if..., What about if you say ... does that still work?)
- use questions that lead to generalisations (e.g., Does it always work?, Can you make connections between ...?, Can you see any patterns?, Can you make connections between...?, How is this the same or different to what we did before?, Would that work with all numbers?).

Through listening, discussing and recording their mathematical thinking, the students were given opportunities to organise and re-organise their mathematical understanding as well as critically evaluate and build on the thinking of their peers.

Focusing on mathematical learning and practices

Students’ mathematical explanations provided the teachers with ways to make on-the-spot assessments of students’ current reasoning and to suggest ways the reasoning could be extended. As the level of student contribution increased, teacher expectations of individuals increased: they required them to contribute more and to become more autonomous in their problem-solving activities. The teachers came to more clearly appreciate the opportunities for learning provided by open-ended, differentiated, “group-worthy” tasks. Group-worthy tasks are problems whose solutions require the perspectives of different students, that can be solved using different methods, and that emphasise important mathematical concepts and principles (Boaler, 2006). A key feature of a group-worthy task is that it is sufficiently challenging and open to allow different students to contribute their ideas.

An example of a group-worthy task

Some teachers at Flatbush School are going to take their children on a trip to the beach. All the children are going in minivans. They all have to have a seat to sit on and one teacher will also need to be in each bus in their own seat. Each minivan has six seats. If there are 48 children going, how many minivans do they need? Do not forget about a seat for a teacher on each bus. What if instead of vans they went in buses that had 26 seats? If there were 178 children going, how many buses would they need?

Focusing on relationships with mathematics

The teachers’ continued interest in using new ways of organising the mathematics lessons, in trialling new activities, and in modelling and pressing students to develop more sophisticated mathematical discourse supported the many changes within the mathematics lessons. The new practices were affirmed by feedback on student progress. Achievement data collected at the beginning and conclusion of each year reflected significant shifts in numeracy strategy stages, matched by general increases in PAT stanines. In addition, the CPF tool helped teachers to be aware of progress in terms of participation and dispositional changes. Specifically, the CPF supported the teachers to address issues of diversity and equity related to the communication and participation patterns in mathematics classrooms, embedded within culturally responsive teaching approaches.

The teachers became more aware that learning about how to relate to mathematics and being a mathematical learner (i.e., developing a positive and productive mathematical disposition) is an important outcome. Dispositions are about ways of thinking and being. Desirable ways of working with mathematics include

confidence, flexibility, perseverance, interest, creativity, appreciation, reflection and monitoring. Not only was the teacher engaging the students' minds; they were also being their advocate and engaging their dispositions for learning (Merz, 2009). Classroom norms that supported the students' development of powerful mathematical practices and dispositions included:

- teachers making their own intellectual dispositions more visible to their students
- repeated expectations that the class continually reflect on and discuss their work
- repeated expectations that the students persevere through confusion or erroneous thinking and continually work towards sense making.

As Mele (aged 10) stated, "It is about working out problems that are challenging and struggling, struggling well, it is to get somewhere further than you are. Struggling is learning."

Major implications

Professional development that focuses on supporting teachers to create learning environments that enable all students to participate as valued and active contributors to the mathematics learning community offers a way to address patterns of underachievement in Māori and Pasifika students. Both the quantitative and the qualitative data illustrated that when diverse students are supported to actively participate in communicating mathematical reasoning, not only do their achievement results significantly increase but they also form powerful dispositions to do mathematics and to think of themselves as mathematicians.

The findings confirm that teacher professional learning and development must do more than build teacher knowledge of mathematical stages and numerical strategies: it must also make explicit the ways they can institute and sustain change in classroom communication and participation. The teachers in this study, like most teachers in New Zealand, had recently participated in national numeracy professional development. Through their participation they had developed a repertoire of mathematical understanding, including knowledge of the strategy stages children work through and a range of strategies to teach students at each strategy stage. Implicit in this professional development was an expectation that teachers would require all students to communicate and develop mathematical explanations of their reasoning. But this was not a key focus, nor were guidelines provided for how teachers could achieve this in their classroom environment.

Making such shifts in classroom communication and participation patterns is challenging. For many teachers it means a disruption to their normal roles, routines and practices as they learn new ways to teach and their students learn new ways to engage in and learn mathematics. The CPF effectively supported the teachers' substantive professional learning. It prompted them to critically examine their pedagogical strategies and classroom practices and provided a way to assess ongoing generative change towards creating a more effective community of mathematical inquiry.

Professional development and a collaborative researcher–teacher relationship were integral to sustaining and extending the initial changes the teachers had made through the national numeracy professional development. Important aspects of this were the regular and ongoing professional development led by the researchers in both out-of-class settings (study groups) and classroom settings (in-class modelling, support and coaching), and the collaborative relationships that developed. The study groups involved a range of activities that provided the teachers with opportunities to construct personalised models of inquiry learning environments and to enlarge their understanding of mathematical practices. This included reading and discussing research articles and watching videos of exemplary teacher actions, discussing and critiquing each others' video-recorded classroom observations, building personal mathematical knowledge, and constructing group-worthy mathematical problems. Such activity illustrated that if we want teachers to develop a set of pedagogical practices that are socially and culturally responsive, they need space and time to reflect on and discuss how they can achieve this.

The CPF was central to both professional development and classroom changes. Fundamental to its use were the opportunities for the teachers to apply the information and skills in their own context. The practical set

of pedagogical actions outlined in the CPF provided the teachers with new ways to orchestrate classroom interactions. Placing a focus on improving student communication and participation in collaborative group interactions raised the teachers' awareness of the need to support their students to interact and engage in mathematical activity. Many students required explicit support to explain, question, justify and engage in "friendly arguing". Likewise, the teachers had to attend to group processes that positioned all students to access the talk.

The teachers drew on their Māori and Pasifika students' concepts of collectivism to develop communal responsibility. As the students gained the confidence to explain and justify their reasoning, both their role in doing mathematics and their relationship with mathematics changed; they assumed an active responsibility to listen, question, explain, justify and take intellectual risks with their reasoning. In turn, as the students grew in confidence to explain and justify, the teachers saw and heard what their students could achieve mathematically. It was clear that supporting productive mathematical dispositions requires more than attention to resources/activities: it also requires that we look closely at the mathematical and cultural environment of the classroom and at the teachers' interpretation of students' mathematical thinking and ways of working on problems. Group-worthy tasks, tools and opportunities to work with others have the potential to support the development of students' positive mathematical dispositions.

References

- Boaler, J. (2006). Promoting respectful learning. *Educational Leadership*, 5, 74–78.
- Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher*, 31(1), 5–8.
- Hunter, R. (2007). *Teachers developing communities of mathematical inquiry*. Unpublished doctoral thesis, Massey University, Palmerston North.
- Macfarlane, A. (2004). *Kia hiwa ra! Listen to culture*. Wellington: NZCER Press.
- Merz, A. (2009). Teaching for mathematical dispositions as well as for understanding: The difference between reacting to and advocating for dispositional learning. *Journal of Educational Thought*, 43(1), 65–78.

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